

# Cambridge O Level

CANDIDATE  
NAME

CENTRE  
NUMBER

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CANDIDATE  
NUMBER

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## ADDITIONAL MATHEMATICS

**4037/12**

## Paper 1

October/November 2023

**2 hours**

You must answer on the question paper.

No additional materials are needed.

## INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

## INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

**Mathematical Formulae****1. ALGEBRA***Quadratic Equation*

For the equation  $ax^2 + bx + c = 0$ ,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial Theorem*

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where  $n$  is a positive integer and  $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

*Arithmetic series*       $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

*Geometric series*       $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

**2. TRIGONOMETRY***Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

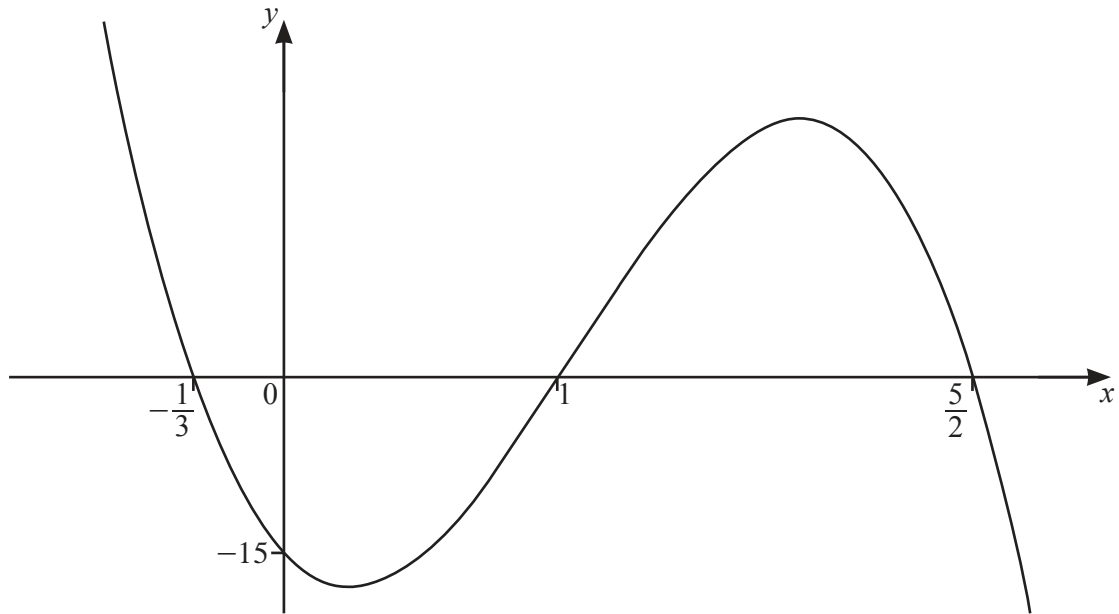
*Formulae for  $\triangle ABC$* 

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

1



The diagram shows the graph of the cubic polynomial  $y = f(x)$ .

- (a) Find an expression for  $f(x)$  in factorised form. Write each linear factor with its coefficients as integers. [3]

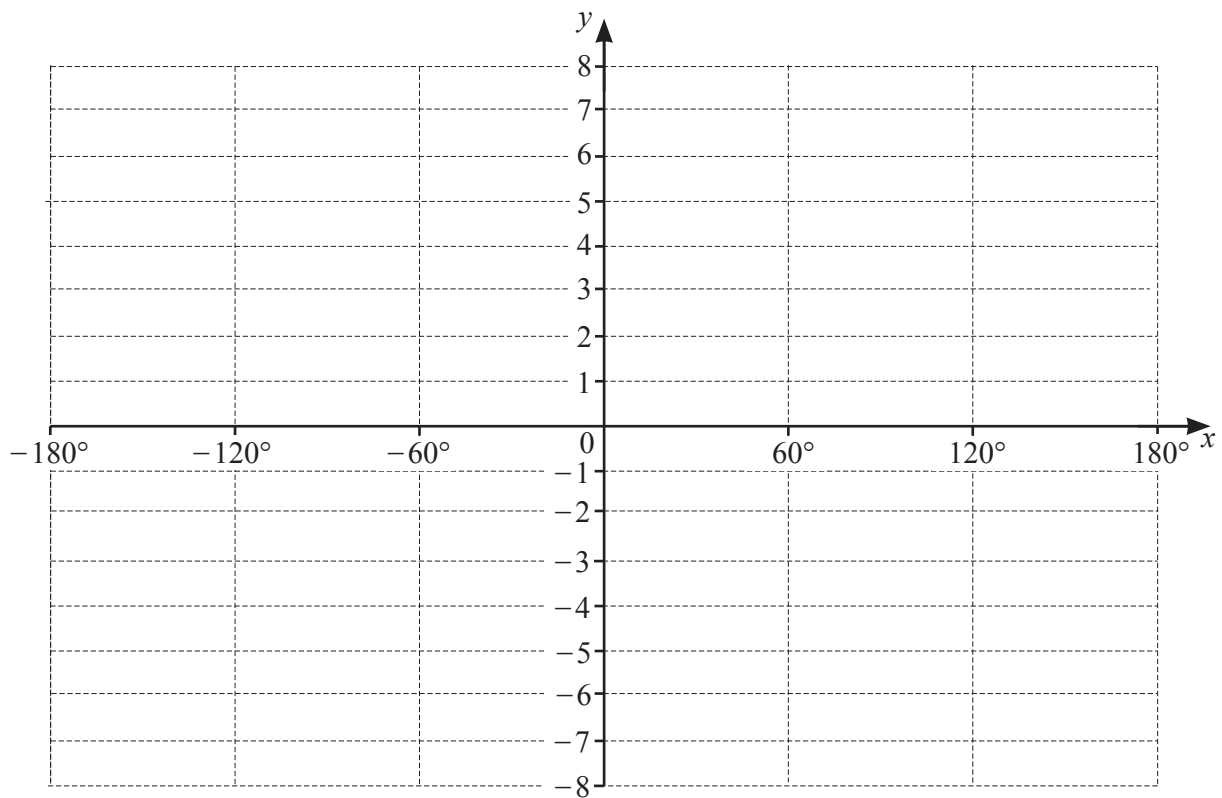
- (b) Write down the values of  $x$  such that  $f(x) < 0$ . [2]

2 The function  $g$  is defined by  $g(x) = 5 \sin \frac{3x}{4} - 2$  for all values of  $x$ .

(a) Write down the amplitude of  $g$ . [1]

(b) Write down the period of  $g$  in degrees. [1]

(c) On the axes, sketch the graph of  $y = g(x)$ , for  $-180^\circ \leq x \leq 180^\circ$ . [3]



- 3 When  $\ln(y+2)$  is plotted against  $x^2$  a straight line graph is obtained. The line passes through the points (2.25, 9.37) and (4.75, 3.92). Find  $y$  in terms of  $x$ . [5]

- 4 (a) It is given that the first four terms, in ascending powers of  $x$ , in the expansion of  $\left(1 - \frac{x}{2}\right)^n$  can be written in the form  $1 - 8x + px^2 + qx^3$ , where  $n, p$  and  $q$  are integers. Find the values of  $n, p$  and  $q$ .  
[5]

- (b) Find the term independent of  $x$  in the expansion of  $\left(\frac{2}{x^2} + \frac{x}{3}\right)^6$ , giving your answer as a rational number.  
[2]

- 5 Solve the equation  $3 \sec^2\left(2\theta + \frac{\pi}{6}\right) = 4$  for  $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ , giving your answers in terms of  $\pi$ . [5]

- 6 The polynomial  $p(x)$  is such that  $p(x) = ax^3 + bx^2 + cx - 5$ , where  $a$ ,  $b$  and  $c$  are integers. It is given that  $p'(0) = 12$ . It is also given that  $p(x)$  has a factor of  $3x - 1$  and a remainder of 95 when divided by  $x - 2$ .

(a) Find the values of  $a$ ,  $b$  and  $c$ .

[7]

(b) Show that the equation  $p(x) = 0$  has only one real root.

[3]



- 7 (a) A 6-digit number is to be formed using the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. Each digit can be used only once in any 6-digit number. A 6-digit number cannot start with 0.

(i) Find how many 6-digit numbers can be formed. [1]

(ii) Find how many of these 6-digit numbers are divisible by 5. [3]

- (b) A committee of 7 people is to be chosen from 6 doctors, 10 nurses and 8 dentists.

(i) Find the number of committees that can be chosen. [1]

(ii) Find the number of committees that can be chosen if all the doctors have to be on the committee. [1]

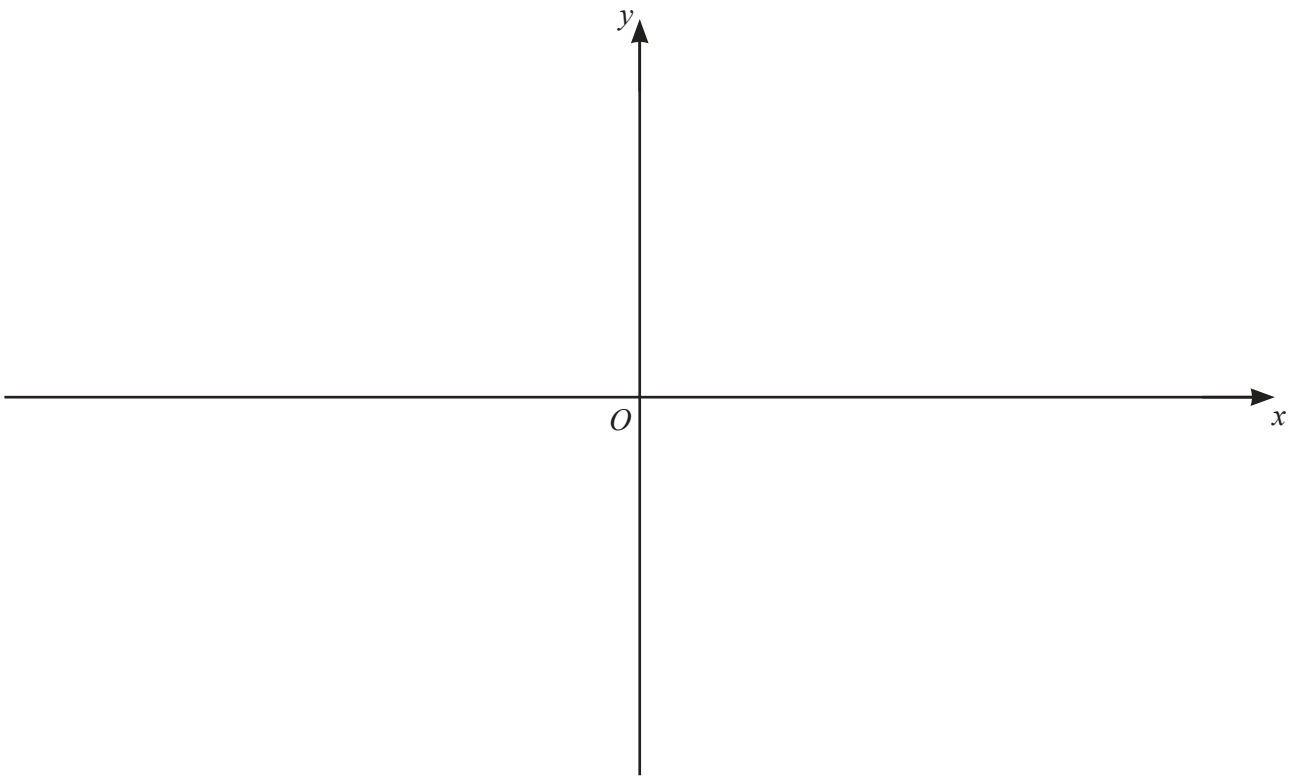
(iii) Find the number of committees that can be chosen if there has to be at least one dentist on the committee. [2]

8 (a) It is given that  $f : x \rightarrow (3x+1)^2 - 4$  for  $x \geq a$ , and that  $f^{-1}$  exists.

(i) Find the least possible value of  $a$ . [1]

(ii) Using this value of  $a$ , write down the range of  $f$ . [1]

(iii) Using this value of  $a$ , sketch the graphs of  $y = f(x)$  and  $y = f^{-1}(x)$  on the axes, stating the intercepts with the coordinate axes. [4]



(b) It is given that  $g(x) = \ln(2x^2 + 5)$  for  $x \geq 0$ ,

$$h(x) = 3x - 2 \text{ for } x \geq 0.$$

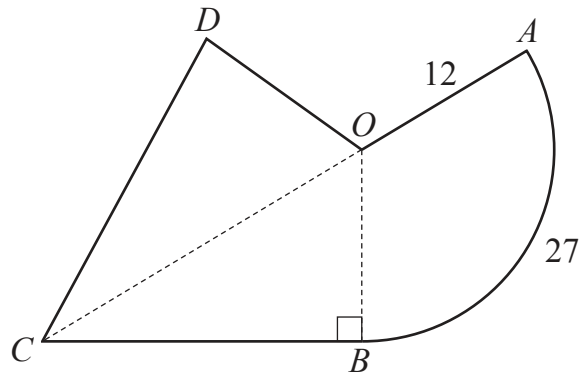
Solve the equation  $hg(x) = 4$  giving your answer in exact form.

[3]

9 Solve the equation  $12x^{\frac{2}{3}} - 5x^{-\frac{2}{3}} - 11 = 0$  for  $x > 0$ . Give your answer correct to one decimal place.

[4]

10 In this question all lengths are in centimetres and all angles are in radians.



The diagram shows a badge which consists of a minor sector,  $OAB$ , of the circle with centre  $O$  and radius 12, and a kite  $OBCD$ , where  $OB = OD$  and  $CD = CB$ . The arc  $AB$  has length 27. The line  $OB$  is perpendicular to the line  $CB$ , and  $COA$  is a straight line.

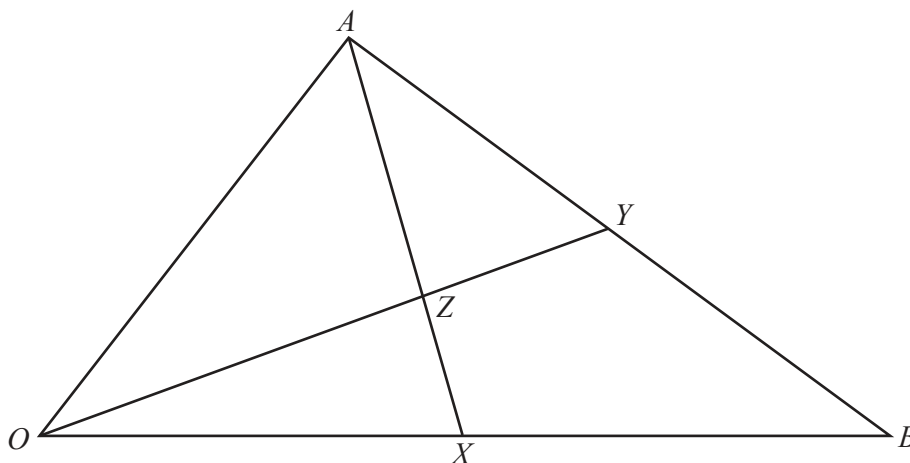
(a) Find the perimeter of the badge.

[4]

(b) Find the area of the badge.

[3]

11



In the triangle  $OAB$ ,  $\overrightarrow{OA} = \mathbf{a}$  and  $\overrightarrow{OB} = \mathbf{b}$ . The mid-point of the line  $OB$  is  $X$ , and the mid-point of the line  $AB$  is  $Y$ . The lines  $OY$  and  $AX$  intersect at the point  $Z$ . It is given that  $\overrightarrow{AZ} = \lambda \overrightarrow{AX}$  and  $\overrightarrow{OZ} = \mu \overrightarrow{OY}$  where  $\lambda$  and  $\mu$  are rational numbers.

(a) Find  $\overrightarrow{OZ}$  in terms of  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\lambda$ . [3]

(b) Find  $\overrightarrow{OZ}$  in terms of  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\mu$ . [2]

(c) Find the values of  $\lambda$  and  $\mu$ .

[3]

(d) Hence find  $\overrightarrow{OZ}$  in terms of **a** and **b** only.

[1]

**Question 12 is printed on the next page.**

12 A curve has equation  $y = \frac{\sqrt{5x-2}}{x-3}$ .

(a) Explain why the curve does not exist when  $x < \frac{2}{5}$ . [1]

(b) Show that  $\frac{dy}{dx}$  can be written in the form  $\frac{-(Ax+B)}{2(x-3)^2\sqrt{5x-2}}$ , where  $A$  and  $B$  are positive integers. [5]

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